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# Robust state-feedback stabilization of discrete-time linear polytopic positive systems using LMIs

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## 1 Introduction

A class of systems that properly represents several real phenomena is called *positive systems*, where the state variables, inputs and outputs, can only assume non-negative values. This study field has some open problems since many of the well-establish results for linear systems cannot be directly applied to positive systems. This abstract proposes a new state-feedback design condition for discrete-time linear polytopic positive systems via the extension of the techniques given in [2] and [3], that use linear matrix inequalities (LMIs) combined with scalar searches.

## 2 Results

Consider the following discrete-time linear polytopic positive system

$$x(k+1) = A(\alpha)x(k) + B(\alpha)u(k) \quad (1)$$

where  $x(k) \in \mathbb{R}^n$  represents the state vector,  $u(k) \in \mathbb{R}^m$  is the control input. The system matrices belong to a polytopic domain, i.e., they can be written as a convex combination of the  $N$  known vertices:  $(A, B)(\alpha) = \sum_{i=1}^N \alpha_i (A_i, B_i)$ ,  $\alpha \in \Lambda$ , and  $\alpha$  is a time-invariant parameter belonging to the unit simplex  $\Lambda = \{\sum_{i=1}^N \alpha_i = 1, \alpha_i \geq 0, i = 1, \dots, N\}$ . Furthermore, all entries of matrices  $(A, B)(\alpha)$  are positive (internally positive system). In order to asymptotically stabilize and guarantee the positiveness of (1) in closed-loop, we consider the following robust state-feedback control law:  $u(k) = Kx(k)$ . The main result of this work is presented next.

**Theorem 1** For given scalars  $\gamma \neq 0$  and  $\varepsilon$ , if there exist matrices  $P(\alpha) = P(\alpha)' > 0 \in \mathbb{R}^n$ ,  $F(\alpha)$  and  $G(\alpha) \in \mathbb{R}^{n \times n}$ ,  $L \in \mathbb{R}^{m \times n}$  and a diagonal matrix  $S \in \mathbb{R}^{n \times n}$ , such that  $A(\alpha)S + B(\alpha)L \geq 0$  and

$$\begin{bmatrix} \begin{pmatrix} A(\alpha)F(\alpha) + \\ F(\alpha)'A(\alpha)' - P(\alpha) \end{pmatrix} & * & * \\ \begin{pmatrix} (A(\alpha)G(\alpha) + \varepsilon B(\alpha)L)' \\ -F(\alpha) \end{pmatrix} & P(\alpha) - G(\alpha) - G(\alpha)' & * \\ (B(\alpha)L)' - \gamma F(\alpha) & \gamma(\varepsilon S - G(\alpha)) & \gamma(S + S') \end{bmatrix} < 0$$

hold  $\forall \alpha \in \Lambda$ , then  $K = LS^{-1}$  assures the positiveness and the closed-loop asymptotic stability of (1).

The proof follows the same steps of Theorem 1 from [3] as well as the procedure to obtain the finite set of LMIs (programmable).

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To illustrate the effectiveness of the technique, consider the following example which analyses the state-space system whose polytopic matrices have the following vertices

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} = \begin{bmatrix} 7.307 & 3.154 & 5.515 & 9.837 & 4.432 & 7.749 \\ 4.725 & 2.438 & 3.764 & 9.150 & 4.266 & 7.402 \\ 11.18 & 5.754 & 10.028 & 11.158 & 5.246 & 9.410 \\ 9.321 & 4.313 & 7.727 & 11.371 & 5.684 & 9.426 \\ 9.287 & 4.289 & 7.688 & 11.632 & 5.665 & 9.567 \\ 11.47 & 5.554 & 9.581 & 6.928 & 3.268 & 5.698 \end{bmatrix}$$

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 4 & 5 \\ 4 & 4 & 5 \\ 5 & 5 & 3 \end{bmatrix}$$

The main goal is to compare Theorem 1 (T1) and other methods from the literature (LMIs (24), (25) and (27) from [1] adapted to handle the stabilization problem). In order to test T1, we consider  $\gamma = -1 \times 10^5$  and the scalar search  $\varepsilon \in [10^{-5}, 10]$ , containing 7 logarithmically spaced values.

All three conditions from [1] were not able to provide stabilizing gains. Considering T1 with the mentioned scalar search, only  $\varepsilon = 10^{-2}$  and  $10^{-3}$  were able to provide feasible solutions, respectively providing the following controllers (truncated with 4 decimal digits)

$$\begin{bmatrix} K(\varepsilon = 10^{-2}) \\ K(\varepsilon = 10^{-3}) \end{bmatrix} = \begin{bmatrix} -2.2316 & -1.0491 & -1.8383 \\ -2.2314 & -1.0485 & -1.8378 \end{bmatrix}$$

These results validate the advantages of employing slack variables (matrices  $F(\alpha)$  and  $G(\alpha)$ ) and scalar searches in order to reduce the conservatism of the synthesis condition at the price of an increase in the computational burden. Finally, note that in Theorem 1 it is not necessary to impose that the Lyapunov matrix is constant or diagonal to obtain robust controllers that guarantee the positiveness and stability of the system.

## References

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